Year: 2009

Role of Λ in the cosmological lens equation

Sereno, M

Abstract: The cosmological constant Λ affects cosmological gravitational lensing. Effects due to Λ can be studied in the framework of the Schwarzschild–de Sitter spacetime. Two novel contributions, which cannot be accounted for by a proper use of angular diameter distances, are derived. First, a term ˆΛ=2mbΛ/3 has to be added to the bending angle, where m is the lens mass and b the impact parameter. Second, Λ brings about a difference in the redshifts of multiple images. Both effects are quite small for real astrophysical systems, ˆΛ 1 arcsec and Δzs 10-7.

DOI: https://doi.org/10.1103/PhysRevLett.102.021301

Other titles: The role of Lambda in the cosmological lens equation

Originally published at:
Sereno, M (2009). Role of Λ in the cosmological lens equation. Physical Review Letters, 102(2):021301.
DOI: https://doi.org/10.1103/PhysRevLett.102.021301

The role of Λ in the cosmological lens equation

Mauro Sereno*
Institut für Theoretische Physik, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland
(Dated: July 29, 2008)

The cosmological constant Λ affects cosmological gravitational lensing. Effects due to Λ can be studied in the framework of the Schwarzschild-de Sitter spacetime. Two novel contributions, which can not be accounted for by a proper use of angular diameter distances, are derived. First, a term $2mb\Lambda/3$ has to be added to the bending angle, where m is the lens mass and b the impact parameter. Second, Λ brings about a difference in the redshifts of multiple images. Both effects are quite small for real astrophysical systems.

PACS numbers: 95.30.Sf, 04.70.Bw, 98.62.Sb

The cosmological constant Λ plays a central role in gravitational physics and observational cosmology, with a fine-tuned $\Lambda \sim 10^{-52}\mathrm{m}^{-2}$ favored by large scale structure observations as a possible choice for dark energy [1]. Λ should take part in all kinds of gravitational phenomena on very different scale-lengths and investigations have been performed on planetary systems [2, 3], gravitational equilibrium of structures [4] and discs orbiting rotating black holes [5]. Actual upper bounds from stellar tests give $\Lambda \lesssim 10^{-42}\mathrm{m}^{-2}$ [3].

The role of Λ in gravitational lensing is still debated. It is well known that Λ and other cosmological homogeneous constituents enter the lens equation through the angular diameter distances [6]. The cosmological lens equation is usually derived combining local results on light deflection, which are based on some asymptotically flat metric which describes the neighborhood of the lens, with considerations on global geometry and angular diameter distances, which are on turn based on the global Friedmann-Robertson-Walker (FRW) spacetime in which the system is embedded [6]. The main contribution of Λ is connected to the propagation of light rays in the nearly homogeneous regions among source, deflector and observer and should be seen as global. A proper use of angular diameter distances [8–10] can then account for the effect of Λ on the process of measurement [11–13]. The fact that the exact differential equation for a light path in the Schwarzschild-de Sitter (SdS) spacetime, i.e. the spherically symmetric Schwarzschild vacuum solution with a cosmological constant, can be written in a form that does not involve Λ [2], differently from other dark energy models [7], further suggests that any local effect should be small. Here, we are interested in local effects of Λ.

The role of Λ in a cosmological gravitational lensing can be properly studied in the framework of the SdS metric [14],

$$ds^2 = f_\Lambda(r)dt^2 - \frac{dr^2}{f_\Lambda(r)} - r^2\left(d\theta^2 - \sin^2\theta d\phi^2\right), \qquad (1)$$

where

$$f_\Lambda(r) \equiv \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right), \qquad (2)$$

and m is the black hole mass. We are using units $G = c = 1$. The SdS metric is a special case of the Mc Vittie metric, which provides an exact description of a point-like lens embedded in a FRW spacetime. In the SdS spacetime, the global expansion is driven only by Λ. The strong advantage of working with the SdS solution is that lightlike geodesics are very well known in those coordinates.

Due to spherical symmetry, photon trajectories can be restricted to the central $\theta = \pi/2$ plane. Let us consider an observer in $\{r_\mathrm{o}, \phi_\mathrm{o} = 0\}$, where ϕ_o has been fixed without loss of generality, and a light source in $\{r_\mathrm{s}, \phi_\mathrm{s}\}$. The orbital equation of a light ray can then be written in terms of the first integral of motion $b(\equiv \dot{\phi} r^2)$ as

$$\phi_\mathrm{s} = \pm \int \frac{dr}{r^2}\left[\frac{1}{b^2} + \frac{1}{r_\Lambda^2} - \frac{1}{r^2} + \frac{2m}{r^3}\right]^{-1/2}, \qquad (3)$$

where the sign of the integral changes at the inversion points in the r-motion; $r_\Lambda(\equiv \sqrt{3/\Lambda})$ is the outer horizon in the de Sitter metric.

We consider the weak deflection limit, where the source and the observer lie in remote regions very far from the lens and photons pass by the lens center at a minimum distance which is much larger than the gravitational radius, i.e. $m/b \equiv \epsilon_\mathrm{m} \ll 1$. In a cosmological scenario, $r_\mathrm{o} \sim r_\mathrm{s} \lesssim r_\Lambda$. Quantities of interest can then be expanded according to the parameters ϵ_m and $\epsilon_\Lambda \equiv b/r_\Lambda$. For the sake of brevity, we will produce our results up to a given formal order in ϵ, collecting terms coming from any combination of the two expansion parameters [8, 15]. Combining such terms is further motivated since $b/r_\mathrm{o} \sim b/r_\mathrm{s} \sim \epsilon_\mathrm{m}$ [16]. The expansion technique is similar to [8] with the main difference that a different expansion scheme is used [24].

The integral in Eq. (3) can then be solved approximately following standard methods and procedures [8, 15, 16]. For $b > 0$, we get

$$\begin{aligned}\phi_\mathrm{s} \ =\ & -\pi - \frac{4m}{b} + b\left(\frac{1}{r_\mathrm{s}} + \frac{1}{r_\mathrm{o}}\right) - \frac{15m^2\pi}{4b^2} - \frac{128m^3}{3b^3} \\ & + \frac{b^3}{6}\left(\frac{1}{r_\mathrm{s}^3} + \frac{1}{r_\mathrm{o}^3}\right) - \frac{2mb}{r_\Lambda^2} - \frac{b^3}{2r_\Lambda^2}\left(\frac{1}{r_\mathrm{s}} + \frac{1}{r_\mathrm{o}}\right) + \mathcal{O}(\epsilon^4).\end{aligned} \qquad (4)$$

Terms of three kinds show up in the azimuthal deflection. Contributions like $(m/b)^i$ represent the usual bending angle by a Schwarzschild lens and the well known higher order corrections. Both $(b/r_i)^i$- and b/r_Λ-factors are geometrical terms. The $(b/r_i)^i$ terms are related to the angles in the observer's sky, $\vartheta \sim b/r_\mathrm{o}$, and in the associated system without the lens, $\beta \sim b/(r_\mathrm{o}+r_\mathrm{s})$; the $(b/r_\Lambda)^i$-factors account for the presence of an outer horizon. The specific form of the geometrical factors in Eq. (4) reflects that the associated system is a FRW spacetime and not the flat Minkowski space. Finally, the term $\delta\hat{\alpha}_\Lambda \equiv 2bm/r_\Lambda^2$ describes the coupling between the lens and the cosmological constant. Neither the source or the observer position enter in $\delta\hat{\alpha}_\Lambda$, which is clearly local. In what follows, we will make the case that such a local coupling should be considered in the lens equation. On a dimensional basis, $\Lambda m b$ is the first dimensionless combination in which all the quantities describing a local bending, i.e. the impact parameter and the lens mass, appear together with the cosmological constant.

The lens equation is a mapping relating the position of the source and the observed position of its images. It is usually given in terms of the apparent angular position of the image in the sky, i.e. the angle ϑ between the tangent to the photon trajectory at the observer and the radial direction to the lens as measured in the locally flat observer's frame [8, 17]. In a cosmological scenario, observer, lens and source are receding. The frame of reference of the moving observer can be oriented parallely to the local flat three-space of the static frame. In terms of the tetrad components of the four momentum P of the photon, $\cos\vartheta = P^{[r]}/P^{[t]}$. Neglecting deviation from the Hubble flow, the motion has to be radial ($v^r = dr/dt \neq 0, d\phi/dt = 0$). In the the SdS metric

$$\sin\vartheta = \frac{\sqrt{1-{v^{[r]}}^2(r_\mathrm{o})}}{1-v^{[r]}(r_\mathrm{o})\sqrt{1-(b/r_\mathrm{o})^2 f_\Lambda(r_\mathrm{o})}}\frac{b}{r_\mathrm{o}}\sqrt{f_\Lambda(r_\mathrm{o})}, \quad (5)$$

with $v^{[r]} = (-g_{rr})/g_{tt})^{1/2} v^r$. The radial motion of a comoving observer expressed in SdS coordinates can be derived by referring to the corresponding Mc Vittie form, where an observer in the Hubble flow has constant spatial coordinates. By means of standard coordinate transformations [18], we get

$$v^{[r]}(r_\mathrm{o}) = \frac{r_\mathrm{o}}{r_\Lambda}\frac{1}{\sqrt{1-2m/r_\mathrm{o}}}. \quad (6)$$

The associated reference spacetime without the lens is crucial in writing the lens equation, since the angular diameter distances as well as the source position β are defined there [19]. The natural choice is the de Sitter metric, obtained by tuning the lens mass to zero and one of the few cases in which the RW metric can be put in a static form [20]. We will consider the spatially flat RW model and the corresponding coordinate transformations to the dS solution [20]. Distances can be easily computed in the associated RW spacetime, and then expressed in SdS coordinates. Since the azimuthal coordinate of the source is not known a-priori by the observer, we have to assume the source to be aligned with the line of sight from the observer to the lens.

The angular diameter distance between a comoving source at z_2 and a comoving observer at z_1 can be written as $D_{12} = r_\Lambda(z_2-z_1)/(1+z_2)$. The distances between the observer and the lens, D_d, the deflector ($r=0$) and the source D_ds and the observer and the source D_s can then be written in terms of radial coordinates plugging in the corresponding redshifts in the associated spacetime, $z_\mathrm{d} = r_\mathrm{o}/r_\Lambda$ and $z_\mathrm{s} = (r_\mathrm{o}+r_\mathrm{s})/(r_\Lambda - r_\mathrm{s})$.

The angle β at which the source would be seen in absence of the lens is also defined in the associated spacetime. In analogy with Eq. (5), β is then given by $\sin\beta = \sin\vartheta(b_\mathrm{s}, m=0)$ with b_s being a fictitious constant of motion which solves the geodesic motion in Eq. (3) for the actual source and observer coordinates but for $m=0$ [8].

Since the lens equation is written in terms of angles, a natural expansion parameter ε is based on the angular Einstein ring, $\vartheta_\mathrm{E} \equiv \sqrt{4mD_\mathrm{ds}/(D_\mathrm{d}D_\mathrm{s})}$: $\varepsilon \equiv \vartheta_\mathrm{E}/(4D)$ [15, 16] where $D \equiv D_\mathrm{ds}/D_\mathrm{s}$. The lens equation is then obtained by writing ϕ_s as a function of either ϑ or β and equating the two expressions [8]. Let us consider source positions $\beta \geq 0$ and an observer radial motion as in Eq. (6). Once we expand the lens equation as a series in ε, the solutions take the form $\vartheta \simeq \vartheta_\mathrm{E}\left\{\theta_0 + \theta_1\varepsilon + \theta_2\varepsilon^2\right\}$ [8, 15, 16]. Up to including terms of order of $\mathcal{O}(\varepsilon^2)$, the image positions ϑ solve the standard lens equation,

$$\beta \simeq \vartheta - D\hat{\alpha}, \quad \hat{\alpha} \simeq \frac{4m}{b_0} + \frac{15\pi}{4}\frac{m^2}{b_0^2}, \quad (7)$$

where the bending angle is the Schwarzschild one up to $\mathcal{O}(\varepsilon^2)$; $b_0 (\equiv D_\mathrm{d}\vartheta)$ is the approximated impact parameter. There are two solutions, ϑ_+ and ϑ_-. Up to this order the cosmological constant enters only through the cosmological distances and there is not any additional contribution to the deflection.

Gravitational coupling effects between the central mass and Λ show up at the next order, giving rise to additional contributions to the deflection that can not be accounted for by using angular diameter distance. In order to illustrate the effect of Λ while still keeping expressions simple, let us consider a source aligned to the line of sight ($\beta = 0$). In this symmetric configuration, a critical tangential circle shows up in the observer's sky, with an angular radius of

$$\begin{aligned}\vartheta_\mathrm{t} &= \vartheta_\mathrm{E}\left\{1 + \frac{15\pi}{32}\varepsilon + \left[4 - \frac{4D^2}{3} - \frac{675\pi^2}{2048}\right.\right. \\ &+ \left.\left.\frac{1}{4}\left(1-\frac{1}{D}\right)\frac{1}{r_{\Lambda\varepsilon}^2} + \frac{1}{r_{\Lambda\varepsilon}}\right]\varepsilon^2\right\}, \quad (8)\end{aligned}$$

where $r_{\Lambda\varepsilon} \equiv r_\Lambda/(4DD_\mathrm{d})$. At this order, the cosmological constant affects the image position. The term $\delta\vartheta_\mathrm{t}^\Lambda = \vartheta_\mathrm{E}\varepsilon^2/(4r_{\Lambda\varepsilon}^2) = (1/4)(D_\mathrm{d}/r_\Lambda)^2\vartheta_\mathrm{E}^3$ comes directly from the azimuthal deflection.

Since the image positions depend on the observer motion, one might as well consider an observer comoving in the associated RW spacetime, where $v^{[r]} = r_\mathrm{o}/r_\Lambda$ or even other radial peculiar motions. Let us then consider a generic radial velocity $v^{[r]} \simeq r_\mathrm{o}/r_\Lambda(1+\delta v^{(2)}\varepsilon^2)$. The corresponding critical circle forms at

$$\vartheta_{\rm t} = \vartheta_{\rm t}(\Lambda=0) + \left\{ \frac{1}{4}\left(1-\frac{1}{2D}\right)\frac{1}{r_{\Lambda\varepsilon}^2} + \frac{1}{4r_{\Lambda\varepsilon}(1-2Dr_{\Lambda\varepsilon})}\left(1+\frac{1-4Dr_{\Lambda\varepsilon}}{2D}\delta v^{(2)}\right)\right\}\varepsilon^2, \tag{9}$$

which reduces to Eq. (8) for a Mc Vittie comoving observer, $\delta v^{(2)} = 4D - 1/r_{\Lambda\varepsilon}$, see Eq. (6). For a particular choice of $\delta v^{(2)}$, the peculiar velocity can null out the effect of Λ. Whereas some contributions to the angular radius depend on the choice of the radial motion, the $\delta\vartheta_{\rm t}^{\Lambda}$ contribution does not depend on $v^{[r]}$.

The choice of the angular diameter distance might hide some other effects. What an observer really measures is the redshift of the source $z_{\rm s}$, which is then plugged in the FRW expression for the distance. The very general formula for the redshift is $1+z_{\rm s} = g_{\alpha\beta}k_{\rm s}^{\alpha}U_{\rm s}^{\beta}/g_{\alpha\beta}k_{\rm o}^{\alpha}U_{\rm o}^{\beta}$, with $k_{\rm o}^{\alpha}$ and $k_{\rm s}^{\alpha}$ the wavevectors of the light ray at the observer and at the source, respectively, and $U_{\rm o}^{\alpha}$ and $U_{\rm s}^{\alpha}$ the four-velocities of the observer and of the source, respectively. Assuming Mc Vittie comoving players, we get

$$z_{\rm s}^{Obs} = \frac{f_\Lambda(r_{\rm o})}{f_\Lambda(r_{\rm s})}\frac{\sqrt{1-\frac{2m}{r_{\rm s}}}+\frac{r_{\rm s}}{r_\Lambda}\sqrt{1-\frac{b^2}{r_{\rm s}^2}f_\Lambda(r_{\rm s})}}{\sqrt{1-\frac{2m}{r_{\rm o}}}-\frac{r_{\rm o}}{r_\Lambda}\sqrt{1-\frac{b^2}{r_{\rm o}^2}f_\Lambda(r_{\rm o})}} - 1. \tag{10}$$

The dependence on the impact factor, which would disappear in absence of Λ, makes the redshifts of the two images different. The difference in the measured redshift of the two images can be written by expressing b in terms of ϑ and then expanding,

$$\Delta z_{\rm s}^{Obs} \simeq \frac{r_{\rm o}}{r_\Lambda}\frac{1+r_{\rm o}/r_{\rm s}}{(1+r_{\rm o}/r_\Lambda)(1-r_{\rm s}/r_\Lambda)}\frac{\vartheta_+^2-\vartheta_-^2}{2}, \tag{11}$$

with $\vartheta_+^2-\vartheta_-^2 \simeq \beta\sqrt{\beta^2+4\vartheta_{\rm E}^2}$. Note that such redshift effect depends on the light ray directions at the source and at the observer and is not linked to the total travel time delay. For $\beta \sim \vartheta_{\rm E}$, the redshift difference is proportional to the square of the Einstein radius ($\propto m$), that is really small. For $\beta \sim \vartheta_{\rm E}$, $r_{\rm o} \sim r_{\rm s} \sim r_\Lambda/2$ and a galaxy cluster lens with $M \sim 10^{15}M_\odot$, $\Delta z_{\rm s} \sim 10^{-7}$.

In our model of universe the light source and the observer are both massless. Due to gravitational redshift, the measured redshift of the deflector will also differ from the associated $z_{\rm d}$. However, redshift measurements are based on spectra which integrate on all of the emitting regions of the lens along the line of sight. Since we do not know the effective radial coordinate we should use in the evaluation of the redshift, it is then safe to still consider for the redshift the associated value. The angular diameter distances based on the measured redshifts, and the corresponding Einstein radius $\vartheta_{\rm E}^{Obs} \equiv (4mD(z_{\rm d}, z_{\rm s}^{Obs})/[D(0,z_{\rm d})D(0,z_{\rm s}^{Obs})])^{1/2}$, differ from the expressions based on the associated ones. It can be easily checked that $\vartheta_{\rm E}^{Obs}$ does not embody the Λ-correction due to the azimuthal deflection.

As a further check we could consider static observers in the SdS spacetime, $dr/dt=0$. In this case the the expression for the distances in terms of radial coordinates are the same used in [8, equations (15-17)]. Up to order $\mathcal{O}(\varepsilon^2)$, the lens equation has still the form of Eq. (7). Up to the next order, the critical circle forms at

$$\vartheta_{\rm t}^{\rm St} = \vartheta_{\rm E}^{\rm St}\left\{1+\frac{15\pi}{32}\varepsilon_{\rm St} + \left[4-\frac{4{D_{\rm St}}^2}{3}-\frac{675\pi^2}{2048} + \frac{1}{4}\frac{1}{r_{\Lambda\varepsilon_{\rm St}}^2} - \frac{\sqrt{1+16{D_{\rm St}}^2 r_{\Lambda\varepsilon_{\rm St}}^2}}{16{D_{\rm St}}^2 r_{\Lambda\varepsilon_{\rm St}}^3}\right]\varepsilon_{\rm St}^2\right\}, \tag{12}$$

where the index St reminds that the distances to be used are those for the static case. The term $\delta\vartheta_{\rm t}^{\Lambda}$ is still there. Equation (12) agrees with [8] via a proper consideration of the different expansion scheme.

We have considered either static or moving observers in a SdS spacetime. For the moving case, we have considered observers comoving either in the Mc Vittie metric or in the associated RW background; effects of peculiar motions have been also discussed. The common outcome is that the local coupling of Λ with the lens mass give rise to an azimuthal shift whose effect can not be embodied by using angular diameter distances. This has been verified considering either distances in the associated RW metric or distances based on the observed redshifts.

After comparing the different results it emerges that the $\delta\hat{\alpha}_\Lambda$ bending has a local origin and can be distinguished form other global contributions, which vary with different assumptions on the distances and the radial motion, and are connected to the presence of an outer horizon in the SdS spacetime. In a general ΛCDM model of universe with dark matter, only the $\delta\hat{\alpha}_\Lambda$ term should be retained, whereas the other global effects of the cosmological constant are embodied by the angular diameter distances. The cosmological lens equation can then be written as

$$\beta \simeq \vartheta - \frac{D_{\rm ds}}{D_{\rm s}}(\hat{\alpha}+\delta\hat{\alpha}_\Lambda),\ \delta\hat{\alpha}_\Lambda \equiv \frac{2mb_0}{r_\Lambda^2} \simeq \frac{1}{2}\vartheta_{\rm E}^2\left(\frac{D_{\rm d}}{r_\Lambda}\right)^2\vartheta. \tag{13}$$

The perturbed image positions are

$$\vartheta \simeq \vartheta_0\left\{1+\frac{D_{\rm d}^2}{2r_\Lambda^2}\frac{\vartheta_0^2}{1+\vartheta_0^2/\vartheta_{\rm E}^2}\right\}, \tag{14}$$

with $\vartheta_0 = (\beta \pm \sqrt{\beta^2+4\vartheta_{\rm E}^2})/2$ the 0-th order solutions. The consequent correction to the critical angular circle is $\delta\vartheta_{\rm t}^{\Lambda} = (1/4)(D_{\rm d}/r_\Lambda)^2\vartheta_{\rm E}^3$. The effect on the observed angles is really small, $\sim \vartheta_{\rm E}^3$. For a source at $z_{\rm s}=1$ behind a lens with $M \sim 10^{15}M_\odot$ at $z_{\rm d}=0.3$ in a standard ΛCDM model, $\delta\vartheta_\Lambda \sim 0.1\mu$arcsec. Note that the local coupling gives rise to an attractive gravitational effect which can not be associated with the repulsive force due to Λ, whose effect is incorporated in the cosmological distances.

Whereas the SdS metric provides a proper framework for the spacetime near the lens, its main shortcoming is that it can not reproduce the shear and focusing due to other matter inhomogeneities. Apart from the very neighbourhood of the lens, such lensing effects are much larger than the deflector action and should be accounted for by using properly modified expressions for the distances [6, 21]. Noteworthily, Λ is supposed to dominate the energy budget of the universe in the very far future so that the SdS spacetime is going to provide a very realistic description of the universe.

Let us now briefly review some previous results prompted by a recent paper [11]. All of the results we are going to discuss seem to be correct, with some apparent disagreement due to different frameworks for interpretation. Even if the geodesic equations in either the SdS metric or the Schwarzschild metric can be formally the same, observers measure Λ-dependent real angles and not coordinate angles [11, 13]. Such an effect can be viewed as an additional contribution $\propto -\Lambda b r_{\rm o}$ to the bending angle, if the lens equation is written in terms of radial coordinates instead of angular diameter distances. The point that such a contribution can be incorporated by the distances, as stated in our Eq. (7), has been made in [8, section 4], where a static observer in a local system was considered, in [9], who perturbatively integrated the null geodesics in the Mc Vittie metric, and in [10], who considered quantities in RW coordinates. However, [8, 10] adopted expansion schemes in which the $\Lambda m b$ term in the geodesic equation discussed here was considered of higher order and then neglected. On the other hand, [9, see equation (30)] likely missed such a term since he considered the distance $D_{\rm ds}$ to be much smaller than the horizon r_Λ, an hypothesis that is correct for local systems but breaks down in a cosmological context.

Approaches adopting the Einstein-Strauss method were also followed. In [22], a SdS vacuole was matched into a FRW background. Assuming that once the light transitions out of the vacuole all the Λ-bending stops, the authors obtained a contribution $-\Lambda b r_{\rm v}/3$, in which $r_{\rm v}$, the radial SdS coordinate of the vacuole boundary, replaces $r_{\rm o}$. As previously discussed, this contribution is related to the distance from the vacuole boundary to the lens center and do not spring from coupling effects so that it should be incorporated in the distance $D_{\rm d}$ from the observer to the lens. Using proper matching conditions, the contribution takes the form of $-(\Lambda b/3)[m/(4\pi\rho_{\rm m})]^{1/3}$: the presence of the cosmological matter density $\rho_{\rm m}$ further suggests that the contribution is not local. Finally, Schucker [23] discussed lensing in the Einstein-Strauss solution with positive Λ integrating the light motion piecewise in a flat FRW solution and in the SdS metric and then pasting the geodesics together at the vacuole radius. In this case the comparison is not easy since a lens equation was not provided and the contribution of Λ to the deflection was not singled out. Furthermore, some higher-order terms were dropped out in the integration in the SdS metric. The $2mb\Lambda/3$ contribution to the bending and the source redshift difference are then novel features of this paper.

M.S. is supported by the Swiss National Science Foundation and by the Tomalla Foundation. M.S. thanks the Queen's University Belfast for the warm hospitality during the final stages of this work.

* Electronic address: `sereno@physik.unizh.ch`

[1] P. J. Peebles and B. Ratra, Rev. Mod. Phys **75**, 559 (2003).

[2] J. N. Islam, Phys. Lett. A **97**, 239 (1983); A. W. Kerr, J. C. Hauck, and B. Mashhoon, Classical Quantum Gravity **20**, 2727 (2003).

[3] E. L. Wright, astro-ph/9805292; M. Sereno and P. Jetzer, Phys. Rev. D **73**, 063004 (2006); P. Jetzer and M. Sereno, Phys Rev. D **73**, 044015 (2006); L. Iorio, Int. J. Mod. Phys. D **15**, 473 (2006).

[4] A. Balaguera-Antolínez, C. G. Böhmer, and M. Nowakowski, Classical Quantum Gravity **23**, 485 (2006); C. Bambi, Phys. Rev. D **75**, 083003 (2007).

[5] P. Slaný and Z. Stuchlík, Classical Quantum Gravity **22**, 3623 (2005).

[6] P. Schneider,J. Ehlers, and E. E. Falco, *Gravitational Lenses* (Springer-Verlag Berlin, 1992); S. Seitz, P. Schneider, and J. Ehlers, Classical Quantum Gravity **11**, 2345 (1994).

[7] F. Finelli, M. Galaverni, and A. Gruppuso, Phys. Rev. D **75**, 043003 (2007).

[8] M. Sereno, Phys. Rev. D **77**, 043004 (2008).

[9] M. Park, Phys. Rev. D **78**, 023014 (2008).

[10] I. B. Khriplovich and A. A. Pomeransky, arXiv:0801.1764.

[11] W. Rindler and M. Ishak, Phys. Rev. D **76**, 043006 (2007).

[12] P. Bakala, P. Čermák, S. Hledík, Z. Stuchlík, and K. Truparová, Central European Journal of Physics **5**, 599 (2007); K. Lake, arXiv: 0711.0673v1.

[13] T. Schücker, arXiv:0712.1559; T. Schücker and N. Zaimen, Astron. Astroph. **484**, 103 (2008).

[14] R. Adler, M. Bazin, and M. Schiffer, *Introduction to general relativity* (New York: McGraw-Hill, 1965).

[15] M. Sereno and F. De Luca, Phys. Rev. D **74**, 123009 (2006).

[16] C. R. Keeton and A. O. Petters, Phys. Rev. D **72**, 104006 (2005).

[17] M. Sereno and F. De Luca, Phys. Rev. D **78**, 023008 (2008).

[18] B. C. Nolan, Classical Quantum Gravity **16**, 1227 (1999).

[19] V. Bozza, arXiv:0807.3872v1.

[20] P. S. Florides, Gen. Relativ. Gravit. **12**, 563 (1980).

[21] R. Kantowski, Astrophys. J. **507**, 483 (1998); M. Sereno, G. Covone, E. Piedipalumbo, and R. de Ritis, Mon. Not. R. Astron. Soc. **327**, 517 (2001).

[22] M. Ishak, W. Rindler, J. Dossett, J. Moldenhauer, and C. Allison, Mon. Not. R. Astron. Soc. **388**, 1279 (2008); M. Ishak, arXiv:0801.3514.

[23] T. Schucker, arXiv:0807.0380.

[24] [8] considered a local system embedded in a region well inside the outer horizon, $r_{\rm o}, r_{\rm s} \ll r_\Lambda$, and decoupled from the global expansion.